

9-5 Study Guide and Intervention

Base e and Natural Logarithms

Base e and Natural Logarithms The irrational number $e \approx 2.71828\dots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

Natural Base e	As n increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\dots$ $\ln x = \log_e x$
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The functions $y = e^x$ and $y = \ln x$ are inverse functions.

Inverse Property of Base e and Natural Logarithms	$e^{\ln x} = x$ $\ln e^x = x$
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Natural base expressions can be evaluated using the e^x and \ln keys on your calculator.

Example 1 Evaluate $\ln 1685$.

Use a calculator.

$$\ln 1685 \approx 7.4295$$

Example 2 Write a logarithmic equation equivalent to $e^{2x} = 7$.

$$e^{2x} = 7 \rightarrow \log_e 7 = 2x \text{ or } 2x = \ln 7$$

Example 3 Evaluate $\ln e^{18}$.

Use the Inverse Property of Base e and Natural Logarithms.

$$\ln e^{18} = 18$$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. $\ln 732$

6.5958

2. $\ln 84,350$

11.3427

3. $\ln 0.735$

-0.3079

4. $\ln 100$

4.6052

5. $\ln 0.0824$

6. $\ln 2.388$

7. $\ln 128,245$

8. $\ln 0.00614$

Write an equivalent exponential or logarithmic equation.

9. $e^{15} = x$

$\ln x = 15$

10. $e^{3x} = 45$

$\ln 45 = 3x$

11. $\ln 20 = x$

$e^x = 20$

12. $\ln x = 8$

$e^8 = x$

13. $e^{-5x} = 0.2$

14. $\ln(4x) = 9.6$

15. $e^{8.2} = 10x$

16. $\ln 0.0002 = x$

Evaluate each expression.

17. $\ln e^3$

3

18. $e^{\ln 42}$

42

19. $e^{\ln 0.5}$

20. $\ln e^{16.2}$

9-5 Study Guide and Intervention *(continued)*

Base e and Natural Logarithms

Equations and Inequalities with e and \ln All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

Example Solve each equation or inequality.

a. $3e^{2x} + 2 = 10$

$$3e^{2x} + 2 = 10$$

Original equation

$$3e^{2x} = 8$$

Subtract 2 from each side.

$$e^{2x} = \frac{8}{3}$$

Divide each side by 3.

$$\ln e^{2x} = \ln \frac{8}{3}$$

Property of Equality for Logarithms

$$2x = \ln \frac{8}{3}$$

Inverse Property of Exponents and Logarithms

$$x = \frac{1}{2} \ln \frac{8}{3}$$

Multiply each side by $\frac{1}{2}$.

$$x \approx 0.4904$$

Use a calculator.

b. $\ln(4x - 1) < 2$

$$\ln(4x - 1) < 2$$

Original inequality

$$e^{\ln(4x - 1)} < e^2$$

Write each side using exponents and base e .

$$0 < 4x - 1 < e^2$$

Inverse Property of Exponents and Logarithms

$$1 < 4x < e^2 + 1$$

Addition Property of Inequalities

$$\frac{1}{4} < x < \frac{1}{4}(e^2 + 1)$$

Multiplication Property of Inequalities

$$0.25 < x < 2.0973$$

Use a calculator.

9.5 Ln e study guide notes

Solve each equation or inequality.

1. $e^{4x} = 120$

$$\frac{\ln 120}{4} = \frac{4x}{4}$$

$$\boxed{1.1969 = x}$$

2. $e^x \leq 25$

$$\ln e^x \leq \ln 25$$

$$x \leq \ln 25$$

$$\boxed{x \leq 3.2189}$$

** take ln of both sides*

3. $e^{x-2} + 4 = 21$

$$e^{x-2} = 17$$

$$\ln 17 = x - 2$$

$$(\ln 17) + 2 = x$$

$$\boxed{x = 4.8332}$$

4. $\ln 6x \geq 4$

5. $\ln(x + 3) - 5 = -2$

6. $e^{-8x} \leq 50$

9.5 Ln e study guide notes

$$7. e^{4x-1} - 3 = 12$$

$$\begin{aligned} & \quad \quad \quad +3 \quad +3 \\ e^{4x-1} &= 15 \\ \ln e^{4x-1} &= \ln 15 \\ 4x-1 &= \ln 15 \\ +1 \quad \quad +1 \\ \frac{4x}{4} &= \frac{(\ln 15) + 1}{4} \\ \boxed{x = .9270} \end{aligned}$$

$$10. 6 + 3e^{x+1} = 21$$

$$8. \ln(5x + 3) = 3.6$$

$$\begin{aligned} e^{3.6} &= 5x + 3 \\ -3 \quad \quad -3 \\ \frac{e^{3.6} - 3}{5} &= \frac{5x}{5} \\ x &= \frac{e^{3.6} - 3}{5} \\ \boxed{x = 6.7196} \end{aligned}$$

$$11. \ln(2x - 5) = 8$$

$$9. 2e^{3x} + 5 = 2$$

$$\begin{aligned} -5 \quad -5 \\ \frac{2e^{3x}}{2} &= \frac{-3}{2} \\ e^{3x} &= -\frac{3}{2} \\ \ln\left(-\frac{3}{2}\right) &= \frac{3x}{3} \\ \text{Not possible} &= x \end{aligned}$$

$$12. \ln 5x + \ln 3x > 9$$