7.1 operations on functions notes

7-1: Operations on Functions

Let f(x) and g(x) be any two functions.

Operation	Definition	Examples if $f(x) = x + 2$, $g(x) = 3x$
Sum	$(f + g)(x) = f(x) + g(x)$	$(x + 2) + 3x = 4x + 2$
Difference	$(f - g)(x) = f(x) - g(x)$	$(x + 2) - 3x = -2x + 2$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	$(x + 2)(3x) = 3x^2 + 6x$
Quotient	$(f/g)(x) = f(x)/g(x)$, $g(x) \neq 0$	$(x + 2)/(3x)$, $x \neq 0$

Example: Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find the following functions.

$(f + g)(x)$

$x^2 + 5x - 2 + 3x - 2$

$\boxed{x^2 + 8x - 4}$

$(f - g)(x)$

$(x^2 + 5x - 2) - (3x - 2)$

$x^2 + 5x - 2 - 3x + 2$

$\boxed{x^2 + 2x}$

$(f \cdot g)(x)$ $f(x) \cdot g(x)$

$(x^2 + 5x - 2)(3x - 2)$

$3x^3 + 15x^2 - 6x - 2x^2 - 10x + 4$

$\boxed{3x^3 + 13x^2 - 16x + 4}$

$(f/g)(x)$

$\dfrac{x^2 + 5x - 2}{3x - 2} \leftarrow$ $x \neq \frac{2}{3}$

$3x - 2 = 0$

$+2 \quad +2$

$3x = 2$

$x = 2/3$

Example: $f(x) = x^2 - 7x + 2$, $g(x) = x + 4$

Find: $(f + g)(x)$ & $(f/g)(x)$

$x^2 - 7x + 2 + x + 4$

$\boxed{x^2 - 6x + 6}$

$\boxed{\dfrac{x^2 - 7x + 2}{x + 4} \quad x \neq -4}$

Composition of Functions: A function is performed and then a second function is performed on the result of the first function.

Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function $f \circ g$ can be described by...

$f(x) = 5x + 2$

$g(x) = 13x - 1$

$[f \circ g](x) = f[g(x)]$

This is read...f of g.

$f[g(x)]$ or $[f \circ g](x)$

***Plug g(x) into f(x).**

$5(13x - 1) + 2$

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Example: If f = {(3,-2), (-1,-5), (4,7), (10,8)}
and g = {(4,3), (2,-1), (9,4), (3,10)}

Find f∘g and g∘f.

$f(g(4)) = f(3) = -2$

$f(g(2)) = f(-1) = -5$

$f(g(9)) = f(4) = 7$

$f(g(3)) = f(10) = 8$

$f \circ g = \{-5, -2, 7, 8\}$

$g(f(3)) = g(-2) =$ undefined

$g(f(-1)) = g(-5) =$ undefined

$g(f(4)) = g(7) =$ undefined

$g(f(10)) = g(8) =$ undefined

Example

#1

A. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = x - 5$ and $g(x) = x^2 + 2x + 3$

$[f \circ g](x) = (x^2+2x+3) - 5 = \boxed{x^2+2x-2}$

$[g \circ f](x) = (x-5)^2 + 2(x-5) + 3$

$x^2 - 10x + 25 + 2x - 10 + 3$

$\boxed{x^2 - 8x + 18}$

B. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = -3$

$[f \circ g](x) = x^2 + 2x - 2$

$[f \circ g](-3) = (-3)^2 + 2(-3) - 2 = 9 - 6 - 2 = 1$

$[g \circ f](x) = x^2 - 8x + 18$

$[g \circ f](-3) = (-3)^2 - 8(-3) + 18 = 9 + 24 + 18 = 51$